Multiple regression:
fitting the best fitting plane

$$\hat{Y} = a + b_1 X_1 + b_2 X_2$$

Where, a = intercept (when X_1=0 and X_2=0)

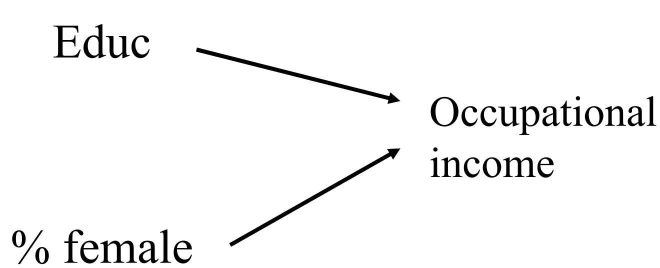
Y = income

X_1 = education

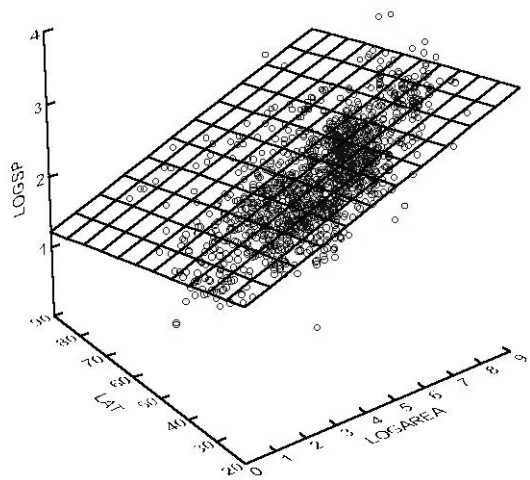
X_2 = % female in occupation

[unit of analysis = occupation]

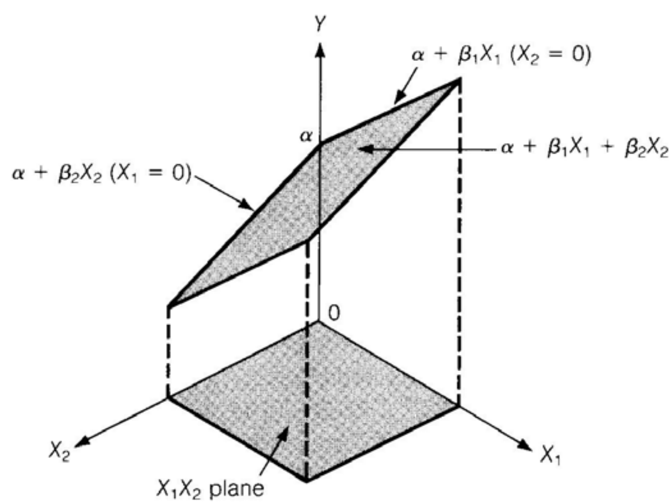
Multiple regression: model



Multiple regression: 3D model



Multiple regression: 3D model



Source: Agresti and Finlay, 1986, p. 317

Multiple regression: interpretation of b's

- b_1 = slope for education (net effect of education on income controlling for percent female; how much income in dollars for each year of education)
- b_2 = slope for % female (net effect of percent female on income controlling for education; how much income in dollars for each percent female)

Multiple regression: interpretation of b's

Coefficients called:

- Metric coefficient
- Net regression coefficient
- Partial regression coefficient
- Unstandardized regression coefficient

Interpretation:

- Net effect
- Independent effect
- Partial relationship
- Controlling for

Multiple regression:
standardized regression coefficients

$$\hat{Y} = a + b_1 X_1 + b_2 X_2 \Rightarrow$$

$$\hat{y} = B_1 x_1 + B_2 x_2$$

where,

$$B_{yx} = b_{yx}(s_x / s_y)$$

Creating standard scores:

$$x_1 = (X_1 - \overline{X_1}) / s_{X_1}$$

Multiple regression:
standardized regression coefficients, or "relative effects"

- B's range from -1 to +1
- Interpretation: a one s.d. change in the independent variable produces a predicted change of "Beta" s.d.'s in the dependent variable, net of other variables
- More common interpretation: if $B_1 > B_2$ then education is more important in predicting income than is % female
- "considerably larger, more than twice as important"

Murdock data:

Y = stratification, X_1=political integration, X_2=money exchange

Society	X_1	X_2	Y	Society	X_1	X_2	Y
001	2	0	1	011	1	1	1
003	3	2	2	013	0	0	0
005	3	3	3	015	2	0	1
007	4	0	2	017	2	3	1
009	0	0	0	019	4	3	2

Calculating standardized and unstandardized coefficients (computational formula)

$$r_{y_i x_i} = \frac{n\sum X_i Y_i - (\sum X_i)(\sum Y_i)}{\sqrt{[n\sum X_i^2 - (\sum X_i)^2][n\sum Y_i^2 - (\sum Y_i)^2]}}$$

Calculating standardized and unstandardized coefficients (adapting for Y and X_2)

$$r_{yx_2} = \frac{n\sum X_2Y - (\sum X_2)(\sum Y)}{\sqrt{[n\sum X_2^{\,2} - (\sum X_2)^2][n\sum Y^2 - (\sum Y)^2]}}$$

Calculating standardized and unstandardized coefficients

$$r_{yx_1} = .865$$

$$r_{yx_2} = .620$$

$$r_{x_1x_2} = .482$$

Calculating standardized coefficients:
general formula

$$B_{yx.z} = \frac{r_{yx} - r_{yz} r_{xz}}{1 - r^2_{xz}}$$

Calculating standardized coefficients:
adapt for $B_{yx1.x2}$

$$B_{yx_1.x_2} = \frac{r_{yx_1} - r_{yx_2} r_{x_1x_2}}{1 - r^2_{x_1x_2}}$$

Calculating standardized coefficients

$$B_{yx_1.x_2} = .737$$

$$B_{yx_2.x_1} = .265$$

$$\hat{y} = .737x_1 + .265x_2$$

Interpretation: political integration is substantially more important than money in determining level of stratification

Calculating unstandardized coefficients

$b_{yx} = B_{yx}(s_y / s_x)$,

using s.d. computational formula

$$s_y = \sqrt{[\sum Y^2 / n] - [\sum Y / n]^2}$$

→ $\overline{Y}$

Calculating unstandardized coefficients

Standard deviations

$s_y = .9 \quad s_{x_1} = 1.37 \quad s_{x_2} = 1.33$

$$b_{yx_1.x_2} = B_{yx_1.x_2}(s_y / s_{x_1})$$
$$= (.737)(.9 / 1.37) = .484$$
$$b_{yx_2.x_1} = B_{yx_2.x_1}(s_y / s_{x_2})$$
$$= (.265)(.9 / 1.33) = .179$$

Calculating intercept

$$\overline{Y} = a + b_1 \overline{X}_1 + b_2 \overline{X}_2$$
$$a = \overline{Y} - b_1 \overline{X}_1 - b_2 \overline{X}_2$$
$$a = (1.3) - (.484)(2.1) - (.179)(1.2)$$
$$a = .069$$

Prediction equation: Interpret!

$$\hat{Y} = .069 + .484X_1 + .179X_2$$

Calculating R^2

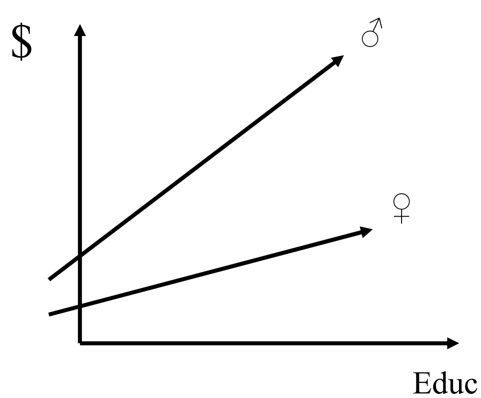
$$R^2{}_{y.x_1x_2} = B_{yx_1}r_{yx_1} + B_{yx_2}r_{yx_2}$$
$$= (.737)(.865) + (.265)(.620)$$
$$= .802$$

<u>Interpretation</u>: 80 percent of the variation in stratification is explained by political integration and money

Standardized vs. unstandardized coefficients

- Use standardized to compare variables ***within*** equations
- Use unstandardized to compare same variable ***across*** equations

Unstandardized coefficients



Caveats

1) Don’t interpret regression lines beyond where you have data
2) Report to three significant nonzero digits (retain larger # of digits in intermediate calculations)
3) Multicollinearity: problem with high correlations